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INTERACTION OF A LIQUID FILM WITH A HIGH-VELOCITY GAS FLOW BEHIND A SHOCK WAVE

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The study of heat- and mass-transfer processes between a liquid film and a turbulent gaseous boundary layer behind a shock wave is important for understanding the mechanism of heterogeneous detonation in the liquid-film–gaseous-oxidizer system. The assumption that the evaporation of the liquid plays the main role [1-5] does not permit describing all the properties of the heterogeneous detonation, calculating the length of the combustion zone, and establishing the effect of the physical properties of the liquid on the parameters of the detonation wave. Under the conditions of a turbulent gaseous boundary layer, large-scale disturbances appear on the gas–liquid interface [6-10], and the surface of the film becomes an irregular structure of waves and splashes, from whose crests drops of liquids are deteched. The drops penetrate into the boundary layer and are carried away by the high-velocity gas flow downstream. Experiments show [6-9] that in the presence of high dynamic velocities of the gas, the mass carried away by the drops is larger than the mass carried away by evaporation. At the present time, there is no justification for neglecting, as in [1-5], the detachment of drops. It was shown in [11] that for Reynolds numbers of the liquid film Re > 100, determined according to the thickness of the film, a system of instability waves, which is capable of completely destroying the film, develops on the surface of the film. In [6], the appearance of developed instability on the interface is described with the help of the dimensionless velocity of the liquid W^+, found from the formula

$$W^+ = m_b/\mu_L. \tag{1}$$

Here $m_b = \int_0^\delta \rho_L u dy$ is the specific mass flow rate of the liquid per unit width of the flow; δ, average thickness of the film; ρ_L, density of the liquid; μ_L, viscosity; u, velocity; and W^+, Reynolds number of the liquid. In the range $90 < W^+ < 150$, a dynamically unstable structure, which approximately corresponds to the transient regime of the flow in the film, develops on the liquid surface. A theoretical analysis of the influence of the detachment of drops on the flow behind the shock and detonation waves has not yet been published in the literature. It is therefore expedient to make use of empirical correlations, based on extensive experimental data. In this paper, we analyze the relative contribution of two different mixture-forming processes behind shock waves and we construct a diagram of the mass-transfer regimes.

We shall study the propagation of a shock wave in the gas–film system in a coordinate system fixed to the wave front (Fig. 1). We formulate the principal assumptions of the problem as follows: 1) the thermophysical properties of the liquid and of the gas are constant; 2) the motion of the liquid and the film is laminar; 3) the turbulent boundary layer in the gas begins immediately behind the wave front; 4) the velocity, enthalpy, and vapor concentration profiles in the gaseous boundary layer are similar; and 5) the wall is adiabatic.

Assumption 2 is valid for very thin films, in which the turbulence appears only at the interface, creating an appearance of "roughness." Assumption 3 is confirmed experimentally [11]. We shall examine processes occurring in direct proximity to the wave front over a time that is short compared to the time for heating the wall, which confirms the validity of assumption 5.

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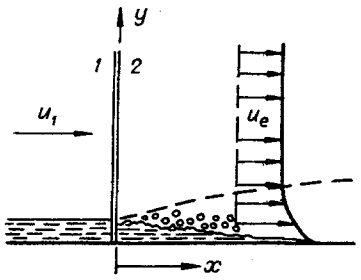

Fig. 1. Coordinate system. 1) Region in front of the shock wave; 2) region behind the shock wave; u_1 is the velocity of the shock wave.

After the passage of a shock wave, a layer in which the liquid, under the action of friction, is entrained by the gas flow into motion, is induced in the film. At a distance x_c from the front, the layer coalesces with the wall and a section of hydrodynamic stabilization of the flow begins. At some distance from the front, a linear velocity profile is established in the film. We shall use the results of [12] to describe the velocity profile in the section 0, x_c

$$u = u_1 - u_2 A \exp[-(\delta - y)/\alpha_e \sqrt{x}].$$

Here $A = (\rho_2/\rho_L)^{2/3}(\nu_2/\nu_L)^{1/3}$; $\alpha_e = (4\nu_L/Au_2)^{1/2}$; ν is the kinematic coefficient of viscosity; and, the indices 1, 2, and L characterize the parameters of the gas in front of the wave, behind the wave, and in the liquid film, respectively. The integral on the right side of (1) is $m_b \approx \rho_L u_2 A \alpha_e \sqrt{x}$. On the section of hydrodynamic stabilization of the fluid flow, the velocity profile is virtually identical to a linear profile, so that here $m_b = 0.5\rho_L u_2 A\delta$. Since the velocity of the film surface is much lower than u_2, the temperature distribution in the liquid is found from the equations for a stationary film

$$u_1 \frac{\partial T}{\partial x} = a \frac{\partial^2 T}{\partial y^2}$$

and from the boundary equations

$$x = 0,\ T = T_1;\ y = 0,\ \frac{\partial T}{\partial y} = 0;\ y = \delta,\ T = T_i.$$

Here a is the coefficient of thermal diffusivity of the liquid; T, temperature; and T_i, temperature of the surface of the film. The solution of the equation with the given boundary conditions has the form [13]

$$T = T_i - 4(T_i - T_1)/\pi \sum_{n=0}^{\infty} \exp(-\lambda_n x/u_1) \sin[\pi(2n+1)(\delta - y)/2\delta]/(2n+1),$$

$$\lambda_n = a[\pi(2n+1)/2\delta]^2.$$

The heat flux into the film is determined as follows:

$$q_i = \lambda \frac{\partial T}{\partial y}\Big|_{y=\delta} = \frac{2\lambda(T_i - T_1)}{\delta \sum_{n=0}^{\infty} \exp(-\lambda_n x/u_1)}. \quad (2)$$

Here λ is the coefficient of thermal condivitity of the liquid. Since the thickness of the film δ and the temperature of the surface T_i in the section x are unknown, the flux q_i is not known. These parameters are calculated from the joining of the solutions in the gaseous turbulent boundary layer and in the liquid film.

Let us examine the processes occurring on the smooth surface of a liquid. The liquid carried away by evaporation is determined, based on assumption 4, from the following equation [14]

$$(\rho v)_v = q_w C_v/(I_2 - I_i)(1 - C_v). \quad (3)$$

Here C_v is the mass concentration of vapor at the surface of the film; q_w, flow of heat from the gas on the surface of the liquid film; and I_2 and I_i, total enthalpy of the gas on the outer boundary of the boundary layer and at the surface of the film. We introduce the coefficient of attenuation of the heat flux due to evaporation of the liquid [14]

$$\psi = q_w/q_{w0} = 1 - \beta f. \quad (4)$$

Here q_{0w} is the heat flux on the nonevaporating surface; $f = (\rho v)_v(I_2 - I_i)/q_{0w}$; $\beta = 0.2 \cdot (M_g/M_v)^{0.1}$ for the turbulent boundary layer; M_g and M_v are the molecular mass of the gas and vapor. The introduction of the coefficient ψ simplifies the analysis, which now reduces to calculating the heat fluxes and friction for the problem without mass transfer. According to Dalton's law, the vapor pressure on the surface of the film p_v, using (3) and (4), is determined from the relation

$$p/p_v = 1 + (1 - \beta f)M_v/fM_g. \tag{5}$$

Here p is the pressure of the vapor–gas mixture in the boundary layer. From the condition for the boundary layer $\frac{\partial p}{\partial y} = 0$, it follows that $p = p_2$. Let the evaporation of liquid be limited by the diffusion of vapor from the surface of the film. Then, it may be assumed that local thermodynamic equilibrium of the vapor–gas–liquid system exists at the surface of the film. This assumption does not contradict the fact that the system is in a nonequilibrium state, but rather that the dissipative processes at the surface of the film are so significant that large deviations from the statistical equilibrium are impossible. In this case, the vapor pressure on the surface of the liquid is close to its equilibrium value, which depends only on the saturation temperature of the vapor or the temperature of the surface of the film. Based on the assumptions stated, we shall assume that $p_v = p_v^s$ (p_v^s is the saturated vapor pressure of the surface). Neglecting the radiation and convective heat flows along the film, from the equation of balance of heat at the interface we find

$$q_w = q_i + (\rho v)_v r. \tag{6}$$

Here r is the specific heat of vaporization of the liquid. Combining (4), (5), and (6), we obtain an equation for the temperature of the surface of the film

$$q_i/q_{0w} = 1 - [\beta + r/(I_2 - I_i)]/[\beta + (p_2/p_v^s - 1)\, M_g/M_v]. \tag{7}$$

To calculate the flux q_{0w}, we shall use Meirl's assumption [5] concerning the validity of Blasius' friction law for the system under study

$$\tau_{0w}/\rho_2 u_2^2 = 0.0225\,(\nu_2/\delta_g u_2)^{0.25}.$$

Here τ_{0w} is the friction stress at the surface in the absence of evaporation; δ_g is the thickness of the boundary layer in the gas. Let the velocity profile in the turbulent boundary layer flow be described by the law [4]

$$(u_i - u)/(u_i - u_e) = \eta^{1/7}, \quad \eta = y/\delta_g,$$

Then

$$\delta_g = (-0.0281 u_2^2 x/u_e^2 \Theta)^{0.8}\,(\nu_2/u_2)^{0.2}, \quad \Theta = \int_0^1 \rho u/\rho_e u_e\,(1 - u/u_e)\,d\eta.$$

Applying Reynolds' analogy, we obtain a relation for the heat flux

$$q_{0w} = \tau_{0w}(I_2 - I_i)/u_2. \tag{8}$$

The instantaneous thickness of the film is determined from the condition that there be no mass flow out of the gas into the liquid. The relations (2), (7), and (8) as well as the specific law for $p_v^s(T)$, permit calculating by the method of successive approximations the temperature of the surface of the liquid film and all the parameters of the problem.

From Eq. (7), we can calculate the maximum temperature to which the liquid film is heated. Indeed, setting $q_i = 0$, we obtain

$$\frac{r}{I_2 - I_{i\max}} = \frac{(p_2/p_{v\max}^s - 1)M_g}{M_v}, \tag{9}$$

where $I_{i\max}$ is the thermodynamic characteristic of the problem.

We shall now analyze the processes occurring on the disturbed surface with the irregular structure of instability waves and detachment of drops. For this, we must introduce a law of friction over the rough surface and use additional information on the validity of Reynolds' analogy. In addition, we must show that the drops of liquid have an insignificant effect on the processes occurring at the surface of the film.

We shall examine the structure of the liquid surface when a shock wave passes in the gas above the liquid. We reduce the stability criterion (1) to the form

$$u_1\delta A/\nu_L < 180\left(\frac{\gamma+1}{2}M_1^2\right)^{1/3}\left(1+\frac{\gamma-1}{2}M_1^2\right)^{2/3}/(M_1^2-1). \quad (10)$$

Here M_1 is Mach's number of the shock wave. In the section $(0, x_c)$, δ must be replaced by $\delta = 9.2\alpha_e\sqrt{x}$. Experimental studies [6-10] show that the structure of the surface of the film changes qualitatively depending on the thickness of the film and the dynamic velocity of the oncoming flow. For very thin films, a smooth surface is characteristic. As δ is increased, long-wavelength low-intensity perturbations, which are later replaced by an irregular three-dimensional structure of short waves and splashed with detachment of drops, appear on the surface. In [16], the measurements of the mean-square (with respect to time) deviations of the disturbed surface of the liquid were found to be correlated with the average thickness of the film for a wide range of Reynolds' numbers of the liquid

$$\Delta = 0.073(\tau_i/\rho_L g)^{0.35}\delta^{0.65}. \quad (11)$$

Here g is the acceleration of gravity; τ_i is the friction stress on the surface of the film on the liquid side, which in the case of a smooth surface is calculated based on Reynolds' analogy

$$\tau_i = \tau_w = \tau_{0w}(1-\beta f).$$

The results in [8] indicate that Reynolds' analogy is also valid for a strongly disturbed surface of the film with detachment of drops. Thus it is necessary to calculate the friction stress on a rough surface in the absence of evaporation. According to [15, 16], the doubled value of the parameter Δ corresponds to Nikuradze's coefficient of equivalent, sandy roughness, i.e., $k = 2\Delta$. Since the criterion (10) determines the region of Re numbers of the liquid in which a developed perturbed structure exists on the surface of the film, the high-velocity gas flow over the film occurs in a state of full manifestation of roughness. This makes it possible to calculate the friction stress using the equations for a rough plate and to find the parameter k from (11). The heat flow can be determined from Reynolds' analogy or from the following considerations. A comparison of the results in [15-17] shows that the ratio of the wavelength of the disturbance to its amplitude at high velocities of the oncoming flow is equal to 0.5-1.5. Thus, to calculate the heat transfer between the turbulent boundary layer and the liquid, it may be assumed that a uniformly distributed, dense "roughness" is present on the surface of the film. Then, according to [18], the heat flow onto the disturbed film of liquid is approximately 1.9 times higher than onto a smooth film.

We shall examine the effect of the drops on the characteristics of a turbulent gaseous layer. The degree to which the drops influence the flow parameters depends on their size, number, and residence time in the boundary layer. To calculate the average diameter of the drops, we shall use the correlations from [19]

$$d_s = 0.5\frac{\sigma_L^{0.6}\rho_L^{0.25}}{\rho_2^{0.85}u_2^{1.2}}\delta^{0.4} + 0.107\left(\frac{\mu_2^2}{\sigma_L\rho_L}\right)^{0.45}\delta^{0.55}.$$

Here σ_L is the coefficient of surface tension of the liquid; d_s is the average diameter of the drops, defined as $d_s = \Sigma nd^3/\Sigma nd^2$, where n is the number of drops and d is their diameter. For waves with $M_1 = 2$ in air, slipping over the surface of a water film 1 mm thick, $d_s \approx 10$ μm. Because of the smallness of their dimensions, the drops move in the flow with velocities close to the velocity of the flow at the corresponding points. The velocity of gravitational settling of fine drops in a high-velocity flow is negligibly small, and the drops rise insignificantly under the action of buoyancy forces. This makes it possible to view the drops of liquid as a passive impurity, distributed in the field of inhomogeneous turbulence. From dimensional analysis [20] it follows that the average vertical velocity of diffusion of impurity particles from the surface of the film is proportional to $\sqrt{\tau_w/\rho_2}$ and is equal to 3-4 m/sec with $M_1 = 2$ in air. It can be shown that with flow high velocities the drops reside for a negligible time near the film and are evaporated primarily far from its surface, without having a significant effect on the heat and mass transfer at the surface of the liquid. The evaporation time of the drops, calculated from the law [21]

$$t = \frac{d_s^2\rho_L}{8\rho_2 D_2 \ln[1+(I_2-I_i)/r]},$$

where D_2 is the coefficient of molecular diffusion, is about 1 msec for water drops with $M_1 = 2$. Over this time, an impurity particle will move beyond the limits of the boundary

layer, before it evaporates. The screening effect of the cloud of drops plays a negligible role in the overall heat balance.

Using the data in [8], we shall examine the process of detachment of the drops behind the wave. The mass carried away from the film with a constant velocity of the oncoming flow is equal to

$$m' = (1+\varepsilon)m_v + e_0 m. \tag{12}$$

Here $m_v = \int_0^x (\rho v)_v dx$; m' is the mass flow due to evaporation and detachment; m is the mass-average velocity with which liquid is injected through the porous wall. The roughness parameter ε characterizes the intensification of the turbulent heat transfer between the boundary layer and the surface of the liquid due to the disturbed conditions of the interface. If the film is hydrodynamically smooth, then $\varepsilon = 0$. The detachment parameter e_0 characterizes the relative rate at which mass is carried away by the drops. It was established in [8] that $\varepsilon = 3/S^{0,8}$, $e_0 = 1 - \exp[-5\cdot10^{-5}S/\sigma_L - 0.05]$, where $S = \sqrt{\rho_2 u_2^2}(T_2/T_1)^{0.25}$. The rate m' was measured at a distance of 25.4 cm from the end of the porous section of the plate. Let us assume that the instantaneous mass flow of liquid, due to the detachment of drops, is proportional to the specific flow rate of the liquid per unit width of the flow in the section x [9]

$$\frac{dm_E}{dx} = B(x)\, m_b.$$

Taking into account (12), we obtain

$$m_E = \int_0^x 1.97\cdot10^{-4}[S/\sigma_L - 1000]\, m_b dx, \tag{13}$$

i.e., $B = 1.97\cdot10^{-4}[S/\sigma_L - 1000]$. The relation (13), valid for an arbitrary section x, together with the stability criterion (10) permits calculating the flow of mass carried away by drops from the surface of the film. The number of drops carried away by the flow from the surface of the liquid is estimated starting from the flow m_E and the average mass of a single drop.

The parameter of the system can be calculated from (7) if the law governing the change of the thickness of the film as a function of the distance from the wave front and the conditions of the problem is known. Such a law is obtained from relations (12) and (13) and the condition that there is no mass flow from the gas into the liquid

$$\begin{aligned} &\text{for} \quad x \leqslant x_c \quad \delta = \delta_i \exp[-1.5Bx - 2(\rho v)_v \sqrt{x}/\rho_L u_2 A\alpha_e], \\ &\text{for} \quad x > x_c \quad \delta = [\delta_c - 2.5(\rho v)_v x/\rho_L u_2 A]/[1 + B(x - x_c)]. \end{aligned} \tag{14}$$

In the derivation of (14), it was assumed that B is virtually independent of x. Relation (14), together with the criterion (10), also permits determining the cross section at which the detachment of drops ceases.

The characteristics of the water-film–air system with different conditions in front of the shock wave were calculated using the method described above. It should be noted that the state of the system in front of the shock wave largely determines the processes occurring behind the wave front. For example, with a high initial temperature T_1 of the system and low pressure p_1, a vapor–gas mixture with a high vapor concentration forms above the film and condenses immediately behind the wave front on the surface of the film [22]. This process will continue until the surface of the film is heated to a temperature corresponding to some intermediate saturated vapor pressure. Further flow of heat from the gas will cause the liquid to evaporate. Since the film is heated slowly behind weak shock waves, the condensation can continue over large distances behind the wave front. In addition, the processes described can also occur on the surface of drops evaporating from the film. Thus when studying systems with a highly volatile liquid or with high initial temperatures, additional difficulties associated with the presence of vapor condensation behind the film appear. Here this process was ignored, since the calculations were performed for water with low initial temperatures.

It follows from Fig. 2 that with other conditions remaining constant, the film is heated more rapidly in the presence of higher initial air pressures due to the intensification of heat exchange at the surface of the film. The increase in the maximum temperature of the

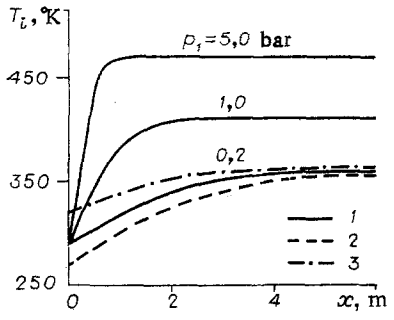

Fig. 2

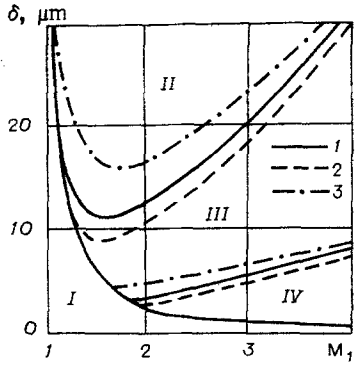

Fig. 3

Fig. 2. Temperature of the surface of a water film behind the shock wave; T_1 = 293.16 (1), 273.16 (2) and 323.16°K (3).

Fig. 3. Diagram of the regimes of mass transfer between the water film and the air flow behind a shock wave; the initial pressure p_1 = 1 atm (the curves are labeled as in Fig. 2).

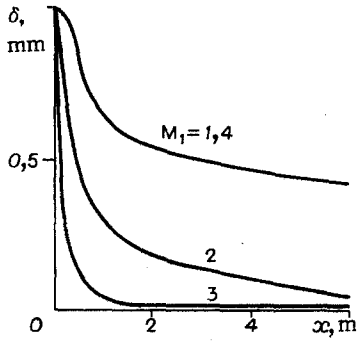

Fig. 4. Variation of the thickness of a water film behind a shock wave; p_1 = 1 atm; δ_1 = 1 mm.

film accompanying the increase in the initial pressure is explained by the balance equation (9). An increase in T_1 reduces the length of the section where the film is heated and leads to some growth of the maximum temperature.

Figure 3 shows a diagram of the regimes of mass transfer between the water film and the air flow behind the wave as a function of the velocity of the shock wave and the thickness of the film. In the region I, the film remains undisturbed, and mass transfer occurs exclusively due to the evaporation of the liquid. In region II, the mass carried away by the drops of liquid ($m_V \leqslant 0.2m_E$) plays the determining role in the mass-transfer process, while in region IV evaporation of the liquid ($m_E \leqslant 0.2m_V$) makes the main contribution to mass transfer. The region III combines the mass transfer regimes in which both processes give comparable contributions. For films with δ = 15-20 μm and higher, for $M_1 \geqslant$ 2-3, the principal mass-transfer mechanism is detachment of drops from the crests of the disturbances on the surface of the film. A change in the initial temperature of the system has virtually no effect on the mass-transfer process at the surface of the liquid (see Fig. 3).

According to (10), the form of the diagram is in many ways determined by the viscosity of the liquid. However, the viscosity of liquids of practical interest is strongly temperature dependent. An analysis of calculations of the heating of the layer of liquid behind the shock wave, as well as the experiments in [7], show that already with M_1 = 2 the difference between the viscosities of the substances has virtually no effect on the behavior of the liquid layer behind the wave. This suggests that the form of the diagram will not change appreciably with a transition to viscous liquids and high numbers M_1. The determining parameters of the problem with high M_1 are the surface tension of the liquid σ_L, ρ_1/ρ_L, p_V^H, δ_1,

and M_1. The diagram in Fig. 3 describes the mass-transfer regimes for a wide class of liquid-film–gas systems with very close properties. It is significant that the viscosity of the liquid in the heating section varies strongly over the thickness of the film and has a determining effect on the formation of the velocity profile in the film. It is therefore expedient to add the coefficient of thermal diffusivity of the liquid to the parameters mentioned above.

Figure 4 illustrates the nature of the variation of the film thickness behind the shock-wave front. Up to the moment of heating, with $M_1 = 3$, the thickness of the film is only 5% of the initial thickness δ_1. The form of the curves in Fig. 4 depends primarily on the initial conditions of the system, which determine the dynamic head of the air flow. The evaporation has little effect on the change in δ and is important only at the last stage, when the film is in region III in the diagram of mass-transfer regimes.

The critical temperature of the surface of a water film is attained only with $M_1 \approx 16$. This result is purely of theoretical interest, because with $M_1 \approx 16$ the film itself is unlikely to exist. The assumption, made by some authors, that the critical temperature of the surface of a fuel film in a system with combustion in the gaseous boundary layer with $M_1 \approx$ 4-5 can be attained requires a detailed study.

The results obtained indicate that in analyzing mass-transfer processes with detonation in a liquid-film–gas system the mass carried off by the liquid from the surface of the film due to friction at the interface separating the phases cannot be neglected. Intense evaporation of liquid drops in a turbulent boundary layer can lead to the creation of a thin fuel-overenriched layer of the mixture near the wall of the pipe and the formation of significant volumes of gaseous mixture far from the wall. Rapid combustion of these volumes will lead to the appearance of the pressure perturbations observed in the experiments.

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